

Supplement to Chapter 2 of *The Science of Digital Media* – Digital Image Representation

Worksheet – Digital Imaging > Aliasing in Sampling¹

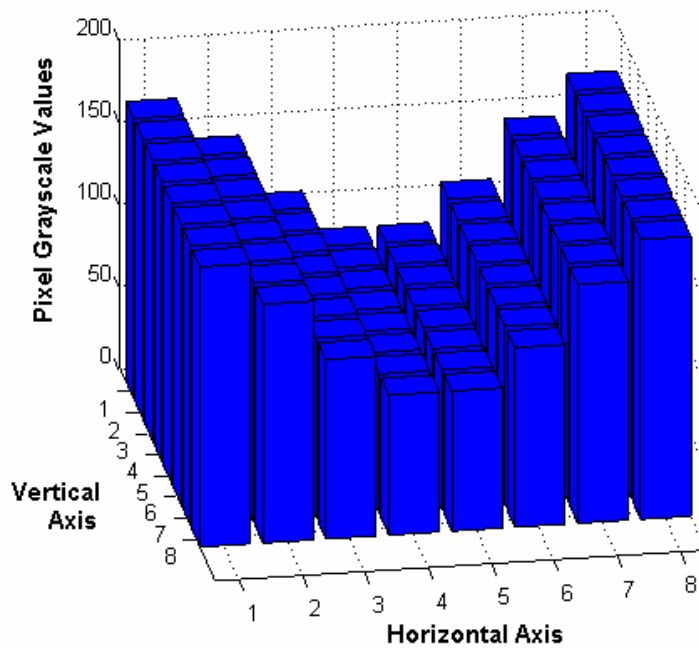
Before completing this worksheet, you should view the on-line interactive tutorial "Aliasing." This tutorial can be accessed at the website for *The Science of Digital Media*.

1. Explain the origin and meaning of the word *aliasing*. What is an aliased wave and how is it different from the original?
2. What does the Nyquist (or sampling) theorem state about the necessary sampling rate in order to avoid aliasing? How can it be applied to complex waves made up of multiple frequency components?
3. Suppose that you are trying to sample a complex wave whose highest-frequency component is 4000 Hz. According to the Nyquist theorem, what is the *minimum* sampling rate necessary to avoid aliasing?

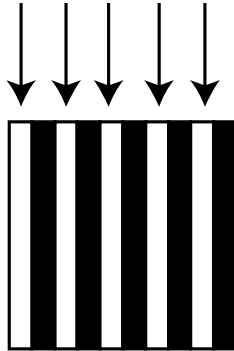
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- The three-dimensional graph below represents an 8×8 block of pixels in a grayscale digital image. The values along the vertical axis represent the pixels' brightness values (0 to 255). Generally speaking, what does the image look like?

Explain how the information displayed in this graph can be used to apply the Nyquist theorem to spatial image data in order to predict and prevent aliasing during sampling. (Note that this figure, generated in MATLAB, numbers the pixel positions from 1 rather than 0.)



5. Suppose that the sampling rate for the striped square below is five horizontal samples for each row of samples taken of the image. Indicate with arrows above or below the box where these five samples could be taken so that the resulting sampled image is completely *white*. Then explain why this sampling rate is not sufficient and the minimum rate necessary to guarantee an accurate sampling.



6. Explain how moiré patterns are formed and how they are related to sampling rates and aliasing. What kind of digital images are most likely to result in moiré effects when sampled?