Supplement to Chapter 2 of *The Science of Digital Media* – Digital Image Representation

Worksheet – Digital Imaging > The CIE Chromaticity Diagram and XYZ Color[1]

Modeling environment: MATLAB

Introduction:

Various ways have been devised to model color so that it can be represented in a computer in three bytes. RGB is a simple model that uses values between 0 and 255 for each of three color channels – red, green, and blue. The choice of "pure" red, green, and blue can vary slightly on different display devices, as long as the three colors are primary relative to each other. This means that no two of them can be combined to create the third.

A disadvantage of the RGB color model is that it isn't possible to create all colors in the visible spectrum by combining positive amounts of red, green, and blue. Different choices of R, G, and B result in different gamuts – ranges of colors that can be represented – but none of these gamuts covers all the visible colors. This was discovered through color matching experiments. In the color matching experiments, people were asked to match all the colors across the visible spectrum as a combination of red, green, and blue primaries. The amount of each primary color needed was recorded and graphed as three functions, $r(\lambda)$, $g(\lambda)$, and $b(\lambda)$, where λ is the wavelength of the color being matched. The results showed that a negative amount of one of the primaries was needed to create some colors. What is meant by a "negative" amount? If it wasn't possible to create a color by a combination of the three primaries, then the experimenters first got as close as possible, and to complete the match they added one of the primaries to the pure color being matched, which is equivalent to subtracting it from the other side.

The purpose of the XYZ color model is to get around this problem of negative amounts of primaries needed to create some of the visible colors. X, Y, and Z are also relatively prime, but the difference is that they themselves don't correspond to visible wavelengths of light. This isn't a problem, since all we care about is that they *combine*, in positive amounts, to create all the visible wavelengths. The same color matching functions were constructed with the X, Y, and Z primaries, and this time the $x(\lambda)$, $y(\lambda)$, and $z(\lambda)$ functions all remained at 0 or above.

The book *The Science of Digital Media* discusses how the X, Y, and Z primaries can be used to create a 2D color space on which various gamuts can be mapped and compared. The purpose of this exercise is to graph the

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functions that lead to the creation of the CIE chromaticity diagram to see with your own eyes how this works.

Exercise 1
Search on the web for a data set of R, G, and B values resulting from the RGB color matching experiments. A variety of data sets are available, varying according to the viewing angle of the participants and the dates when the data was collected. Two examples are the Stiles and Burch (1955) 2-degree RGB chromaticity coordinates and the Stiles and Burch (1959) 10-degree RGB chromaticity coordinates.

Graph these data values to see if they match the graphs in the book.

Exercise 2
Find similar data points for the XYZ color matching functions and graph these.

Exercise 3
Use the X, Y, and Z values that you used in the previous exercise to create the 3D CIE color space. This is done by plotting $(x(\lambda), y(\lambda), z(\lambda))$ as points in 3D space. To match the figures in the book, let the Y-axis be vertical, the X-axis be horizontal, and the Z-axis come out the page.

Exercise 4
Draw the outline of the $X + Y + Z = 1$ plane on this same figure(s).

Exercise 5
Project the $(x(\lambda), y(\lambda), z(\lambda))$ parametric surface onto the $X+Y+Z = 1$ plane.

Exercise 6
Now, in a separate 2D figure, project this result onto the XY plane.

Exercise 7
Draw the RGB gamut on this plane, given the following:

	R	G	B
x	0.64	0.30	0.15
y	0.33	0.60	0.06