Supplement to Chapter 1 of *The Science of Digital Media* – Digital Data Representation and Communication

Worksheet – A Digital Image as a Waveform[1]

Modeling environment: MATLAB and an image processing environment

Introduction:

It's fairly easy to think of sound as a wave. Thinking of an image as a wave is not as intuitive, in part because an image "wave" exists in two dimensions. But representing both sound and images as waveforms is fundamental to digital media, since it facilitates the mathematical operations for editing and compression.

Mathematically, a one-dimensional wave can be defined by a sinusoidal function – a function whose values go up and down smoothly at some frequency – a function like $y = sin(x)$, $y = 3sin(2.5x)$, and so forth. We're accustomed to thinking of sound as a wave, created from rising and falling air pressure that results from vibrations. If the sound is a pure tone, it can be represented by a single-frequency sinusoidal function. Most sounds are much more complex, much more irregular in their ups and downs. This presents no particular difficulty in how we manipulate them mathematically, however, because a complex wave is in fact nothing more than a sum of sinusoidal frequency components.

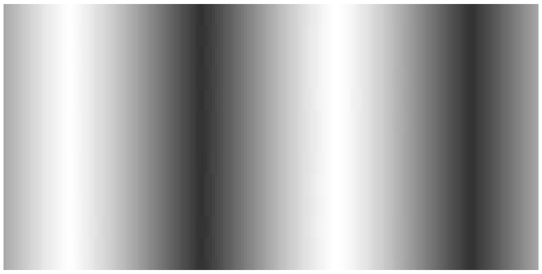
An image can be thought of as a two-dimensional waveform. That is, each horizontal row of pixel values in the image defines a waveform, as does each vertical column. If you put these rows and columns together, they define a surface, which we might call a two-dimensional waveform. The value of representing images by means of waveforms – i.e., combinations of sinusoidal functions – is that we can more easily do mathematical operations on the images, thus editing and compressing them.

The purpose of this worksheet is to allow you to see for yourself how the pixel values from a digital image define a waveform, first in one dimension, and then in two.

Exercise 1

Create a digital image that looks like this:

[1]This material is based on work supported by the National Science Foundation under Grant No. DUE-0340969. This worksheet was written by Dr. Jennifer Burg (burg@wfu.edu).



Hints: In Photoshop, you can create a gradient from white to black and then copy and paste it repeatedly. The gradient above is shifted so that it starts in the middle gray. (We've done it this way so that when a line of the gradient is graphed, it looks like a sine wave.) Be sure the image is in grayscale mode and save the file in an uncompressed form, like BMP or TIFF, both of which are recognized by MATLAB's *imread* function.

Exercise 2

Import the image you just made into MATLAB. Display it in MATLAB.

Hints: Be sure that you're in the right directory in the MATLAB environment. Use *imread* and *imshow* to read the image and display it. Give the image a variable name, like *im*. Then you can look at the data values by simply typing *im*.

Exercise 3

From the image you just created, create another one that is just one horizontal line of pixels from the first image you created. Import this image into MATLAB. Call the imported image *imline*.

Hints: In Photoshop, you can use the crop tool, setting the height to one pixel. Save this as a new image. Import it into MATLAB, calling it with *imline*

Exercise 4

In MATLAB, graph the data in *imline*.

Hints: *imread* brings the image data from *imline* into MATLAB anl array.

Then convert the values to type *double* and shift then so that they are centered on 0, converting
imline = double(imline) – 128;
Now you have an array of values between -128 and 127. You can graph these with
plot(imline);

You can see that one horizontal line of pixel values defines a waveform. The "wave" is not entirely smooth because a digital image defines a set of discrete values. Still, you can see the essential waveform. We had you shift the values by -128 degrees so that the graph would seem more like a sine wave, centered on $y = 0$.

Exercise 5

Graph the data from the full image, *im*, using *surf* (which produces a graph of the surface).

Note that you have just a finite set of discrete values defining this image. It is, after all, a digital image. But the graphing function smooths it out for you.

The pixel values – originally between 0 and 255 – define a surface above and below the *x,y* plane. The higher the value, the higher the point off the plane. In this sense, an image defines a two-dimensional wave.